PHYS 399, Homework #5
Due Wednesday, June 3, start of class.

Key Concepts

- Fourier Series
- Discrete Fourier Transform

Reading: Fourier Handout

Homework Problems

1. Use MATLAB to produce a plot of the Fourier Series expansion for the step function (Eqn. 5 in the handout) for the first five non-constant terms $n = 1, 3, 5, 7, 9$.

2. Find (by integration) the real Fourier coefficients a_n and b_n for a rectified sine function given by

$$f(x) = \begin{cases} -sin(x), & -\pi < x < 0, \\ +sin(x), & 0 < x < +\pi. \end{cases}$$

   What is the lowest non-constant term in the Fourier series? In other words, what is the fundamental frequency of this rectified sine function compared to the sine function itself?

3. Write a MATLAB function which will produce a vector of values sampled at discrete times from the function $A\sin(2\pi\nu t) + B$, where A, B, ν, as well as the sampling frequency ν_s and N (the total number of samples) are input parameters. This is to create "fake data" which we can use in the next problem. Be careful to make sure your discrete time values have a spacing of exactly $\Delta t = 1/\nu_s$. Demonstrate that your function works by producing a plot with $A = 1$, $B = 1$, $\nu = 1$, $\nu_s = 20$ and $N = 100$.

4. Use your creation from the last problem to explore aliasing. Generate a curve with $N = 100$ where $\nu = 9.9$ and $\nu_s = 10$. What is the apparent frequency from the plot and how does this compare to the true frequency ν? Explain qualitatively what is going on here. What minimum sampling frequency ν_s would you need here to avoid aliasing?

5. Write a MATLAB function which will plot the amplitude vs. frequency for a time series of data (like the output of problem 3). In addition to your data, this function will need to know the $\Delta\nu = \nu_s/n$ in order to properly plot amplitude vs. frequency. Check that this works by applying this function to the output of problem 3. Attach the plot of amplitude vs. frequency as well as your code. If you are feeling particularly lazy, you can use `fftshift` and leave the answer in terms of $\pm\nu$ rather than fixing it to give amplitudes only for $0 < \nu < \nu_c$. Compare the amplitudes seen with what you would expect.

6. Take the Fourier transform of the same function with $\nu = 2$ rather than $\nu = 1$. Attach (or just sketch) the amplitude vs. frequency and explain why you see what you see.

7. Do the same thing for $\nu = 2.05$. What is different here? How can we understand this?

8. Redo the last problem but now use $N = 1000$ rather than $N = 100$. Explain in words what happened. What is the difference in the frequency range sampled by this larger value of N? What is the difference in the frequency resolution?