PHYS 399
HW # 4

10.11

a) $p = 1/6, \quad n = 5 \qquad \nu = 0, 5$

$$P_{n,p}(\nu) = \frac{n!}{\nu!\,(n-\nu)!}\, p^{\nu}\,(1-p)^{n-\nu}$$

ν	P	
0:	$\frac{5!}{5!}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5$	$= 40.2\%$
1:	$\frac{5!}{1!\,4!}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4 = \left(\frac{5}{6}\right)^5$	$= 40.2\%$
2:	$\frac{5!}{2!\,3!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = 10\cdot\frac{5^3}{6^5}$	$= 16.1\%$
3:	$\frac{5!}{3!\,2!}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 = 10\cdot\frac{5^2}{6^5}$	$= 3.2\%$
4:	$\frac{5!}{4!\,1!}\left(\frac{1}{6}\right)^4\frac{5}{6} = 25\cdot\left(\frac{1}{6}\right)^5$	$= 0.3\%$
5:	$\left(\frac{1}{6}\right)^5$	$= 0.01\%$

b) 3 or more $= P(3) + P(4) + P(5) = 1 - P(0) - P(1) - P(2)$

$= 3.5\%$

c) 5 aces $= \left(\frac{1}{6}\right)^5$

Any 5 of a kind $= 6\left(\frac{1}{6}\right)^5 = 0.08\%$

10.14

$$\bar{\nu} = \sum \nu B_{n,p}(\nu)$$

Starting w/ 10.5

$$(p+q)^n = \sum_\nu \binom{n}{\nu} p^\nu q^{n-\nu}$$

$$\frac{d}{dp} = \sum_\nu \binom{n}{\nu} \nu p^{\nu-1} q^{(n-\nu)} = n(p+q)^{n-1}$$

For real probabilities $p+q=1$, multiply both sides by p

$$\sum_\nu \binom{n}{\nu} \nu p^\nu q^{(n-\nu)} = np(1)^{n-1}$$

$$\sum \nu B_{n,p}(\nu) = np$$

10.16

ν	$B_{4,\frac{1}{2}}(\nu)$	Gauss $\mu = np = 2$, $\sigma = \sqrt{np(1-p)} = 1$	Δ
0	$\left(\frac{1}{2}\right)^4 = 6.25\%$	$39.9\% \, e^{-2} = 5.40\%$	-0.8%
1	$4 \cdot \left(\frac{1}{2}\right)^4 = 25.0\%$	$39.9\% \, e^{-\frac{1}{2}} = 24.2\%$	-0.8%
2	$\frac{4 \cdot 3}{2}\left(\frac{1}{2}\right)^4 = 37.5\%$	$39.9\% \, e^{0} = 39.9\%$	$+2.4\%$
3	$= B(1) = 25.0\%$	$= 24.2\%$	-0.8%
4	$= B(0) = 6.25\%$	$= 5.4\%$	-0.8%

$$\boxed{\text{Good to} \sim 1\%}$$

$$G(\nu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\nu-2)^2}{2}} = (39.89\%) \, e^{-\frac{(\nu-2)^2}{2}}$$

10.18

18 or more heads in 25 tosses.

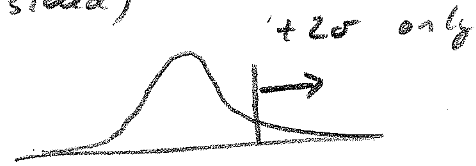
Use gaussian $\mu = np = 12.5$

$\sigma = \frac{1}{2}\sqrt{n} = 2.5$

Use $\nu \geq 17.5$

$$\frac{(17.5 - 12.5)}{2.5} = +2\sigma$$

$P(\text{above}) = [1 - P(<2\sigma)]/2$ (single sided)



$= 2.3\%$

11.3

$n = 5.0 \times 10^{19}$ $p = 3.0 \times 10^{-20} / 5$ sec.

a) Mean rate $\mu = np = 1.5$ decays / 5 sec interval

b)

ν	$P_{1.5}(\nu) = e^{-1.5}(1.5)^{\nu}/\nu!$
0	22.3%
1	33.5%
2	$22.3 \cdot (1.5)^2/2 = 25.1$
3	$22.3 \cdot (1.5)^3/6 = 12.5$

c) 4 or more is $1 - \sum_0^3 P(\nu) = 6.6\%$

11.9) Follow the logic in problem 11.6 b)

$$1 = \sum_{\nu} e^{-\mu} \mu^{\nu}/\nu!$$

Differentiate to get

$$0 = \sum\left(-e^{-\mu} \mu^{\nu}/\nu! + e^{-\mu}\, \nu\, \mu^{\nu-1}/\nu!\right)$$

And again

$$= \sum\left(e^{-\mu}\mu^{\nu}/\nu! - \nu e^{-\mu} \mu^{\nu-1}/\nu! - e^{-\mu}\nu \mu^{\nu-1}/\nu! + \nu(\nu-1)\, e^{-\mu} \mu^{(\nu-2)}/\nu!\right)$$

Multiply by μ^2 and collect

$$= \mu^2 \sum \underbrace{e^{-\mu}\mu^{\nu}/\nu!}_{1} - 2\mu \sum \nu\, e^{-\mu} \mu^{\nu}/\nu! + \sum \nu(\nu-1)\, e^{-\mu} \mu^{\nu}/\nu!$$

$$= \mu^2 - 2\mu\bar{\nu} - \bar{\nu} + \sum \nu^2 e^{-\mu} \mu^{\nu}/\nu!$$

So the thing we want $\overline{\nu^2} = \bar{\nu} + 2\mu\bar{\nu} - \mu^2$

using $\bar{\nu} = \mu$

$$\overline{\nu^2} = \mu + 2\mu^2 - \mu^2 = \mu + \mu^2$$

b) So $\sigma_{\nu}^2 = \overline{\nu^2} - \bar{\nu}^2 = \mu^2 + \mu - \mu^2 = \mu$

So $\sigma_{\nu} = \sqrt{\mu}$

11.10

Rate 20/min

Want $\frac{\delta R}{R} = 0.04$ need $\frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = 0.04$

$N = 625$ $\quad 625/20 \cdot \text{min} = 31.25$ min.

11.20

$N_{tot} = 225$ in 10 min $\quad R_{tot} = \frac{225 \pm 15}{10}$

$= (22.5 \pm 1.5)$ counts/min

$N_{bgd} = 90$ in 6 min

$R_b = \frac{90 \pm 9}{6}$

$= (15.0 \pm 1.6)$ counts/min

Activity $R_{tot} - R_b = (7.5 \pm 2.2)$ counts/min (Kept one extra digit here)

$= (450 \pm 130)$ counts/hr

This is a $3.5\,\sigma$ excess over background, which would have a very low ~0.02% chance of happening from the background alone.