PHYS 399
HW #3

7.1

Value	Error	Weight	$w_i x_i$
1.4	0.5	4	5.4
1.2	0.2	25	30.0
1.0	0.25	16	16.0
1.3	0.2	25	32.5

$$\sum w_i x_i = 83.9$$

$$\sum w_i = 70$$

$$x_{wav} = 1.20$$

$$\sigma_{wav} = \frac{1}{\sqrt{\sum w_i}} = 0.12$$

$$\boxed{x = 1.20 \pm 0.12}$$

7.7

$$x_{wav} = \frac{\sum w_i x_i}{\sum w_i} \qquad w_i = \frac{1}{\sigma_i^2} = \frac{1}{\sigma^2} \text{ (const)}$$

$$= \frac{\frac{1}{\sigma^2}\sum x_i}{\frac{1}{\sigma^2}\sum 1} = \frac{\sum x_i}{N}$$

$$\sigma_x^2 = \frac{1}{\sum w_i} = \frac{1}{1/\sigma^2 \sum 1} = \frac{\sigma^2}{N}$$

$$\sigma_x = \sigma/\sqrt{N} \quad \text{as usual}$$

7.8/

$$X_{wav} = \frac{\Sigma w_i x_i}{\Sigma w_i}$$

The assumption is that the weights have no uncertainty (are exact coefficients)

$$\left[\delta\left(\Sigma w_i x_i\right)\right]^2 = \Sigma\left[\delta(w_i x_i)\right]^2$$ — errors on sum add in quadrature

but $\delta(w_i x_i) = w_i \, \delta x_i = \frac{\sigma_i}{\sigma_i^2} = \frac{1}{\sigma_i}$

So $\left[\delta\left(\Sigma w_i x_i\right)\right]^2 = \Sigma \, 1/\sigma_i^2 = \Sigma w_i$

Now the denominator is just a constant, or:

$$\frac{\delta X_{WAV}}{X_{WAV}} = \frac{\delta\left[\Sigma w_i x_i\right]}{\Sigma w_i x_i}$$

So $\delta x_{WAX} = X_{WAV} \frac{\delta\left[\Sigma w_i x_i\right]}{\Sigma w_i x_i}$

$$= \frac{\Sigma w_i x_i}{\Sigma w_i} \frac{\sqrt{\Sigma w_i}}{\Sigma w_i x_i}$$

$$= \frac{1}{\sqrt{\Sigma w_i}}$$ (eqn. 7.12)

Prob. 4

There is clearly no "right" way to do this. My solution was:

```
function [ avg, err ] = wavg( filename )
%WAVG Summary of this function goes here
%   Detailed explanation goes here

data = dlmread(filename);

% Probably a better way to do this, but get x and y vectors
xvec = data(:,1:1);
yvec = data(:,2:2);
wt = 1./(yvec.*yvec);

err = 1/sqrt(sum(wt));
avg = sum(wt.*xvec)/sum(wt);

end
```

From this I get:

7.1: 1.20 ± 0.12

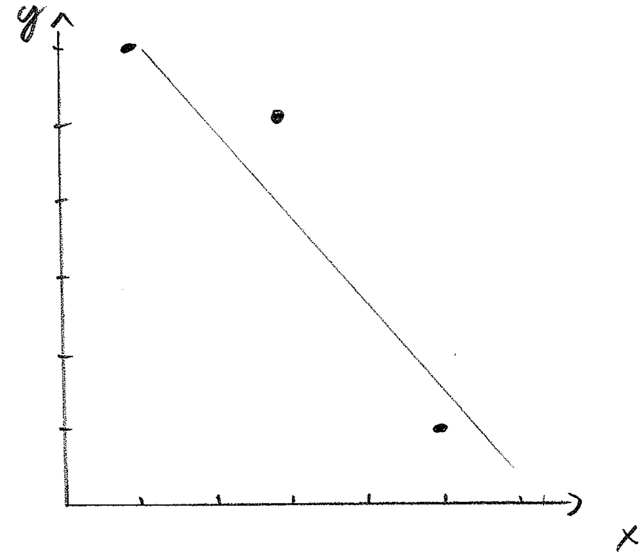
7.2: (1968.1 ± 0.8) MeV/c^2

7.4: (500 ± 6) nm

Make sure you have the right sig. digits here!

8.1

$y = A + Bx$



x	y	x^2	xy
1	6	1	6
3	5	9	15
5	1	25	5

So $\sum x^2 = 35$

$\sum x = 9$

$\sum xy = 26$

$\sum y = 12$

$$\Delta = N\sum x^2 - (\sum x)^2 = 3 \cdot 35 - 81 = 24$$

$$A = \frac{\sum x^2 \sum y - \sum x \sum xy}{\Delta} = \frac{35 \cdot 12 - 9 \cdot 26}{24}$$

$$= 7.75$$

$$B = \frac{N\sum xy - \sum x \sum y}{\Delta} = \frac{3 \cdot 26 - 9 \cdot 12}{24}$$

$$= -1.25$$

y intercept is at

$0 = A + Bx \qquad x = -A/B$

$= 6.2$

Compared to polyfit:

This gives exactly the same answer!

8.5/ $y = Bx$

$$\chi^2 = \sum \frac{(y_i - Bx_i)^2}{\sigma_y^2}$$

$$\frac{\partial \chi^2}{\partial B} = \frac{1}{\sigma_y^2}\sum\left[-2x_i(y_i - Bx_i)\right]$$

$$= -\frac{2}{\sigma_y^2}\sum(x_i y_i - B x_i^2)$$

$$= 0$$

$$\Rightarrow \quad \sum x_i y_i = B\sum x_i^2$$

$$B = \frac{\sum x_i y_i}{\sum x_i^2}$$

8.10/ Next page

8.18/

$$\frac{\delta B}{B} = \frac{\delta[\sum x_i y_i]}{\sum x_i y_i}$$

x_i has no uncertainty by assumption

$$\delta B = \frac{\delta[\sum x_i y_i]}{\sum x_i^2} \leftarrow \text{constant}$$

So: $$(\delta[\sum x_i y_i])^2 = \sum(\delta(x_i y_i))^2 = \sum x_i^2(\delta y_i)^2$$

Now $$(\delta y_i)^2 = \frac{1}{N-1}\sum(y_i - Bx_i)^2 = \sigma_y^2 \text{ (const.)}$$

$$\sum x_i^2 \sigma_y^2 = \sigma_y^2 \sum x_i^2$$

So: $$\delta B = \frac{\sqrt{\sigma_y^2 \sum x_i^2}}{\sum x_i^2} = \frac{\sigma_y}{\sqrt{\sum x_i^2}}$$

8.10/ I will do this by hand

x	y	σ_y	w	wx	wx^2	wxy	wy
1	2	0.5	4	4	4	8	8
2	3	0.5	4	8	16	24	12
3	2	1	1	3	9	6	2
			9	15	29	38	22

$$\Delta = \Sigma w \, \Sigma wx^2 - (\Sigma wx)^2 = 9 \cdot 29 - 15^2 = 36$$

$$A = \frac{\Sigma wx^2 \, \Sigma wy - \Sigma wx \, \Sigma wxy}{\Delta} = \frac{1}{36}(29 \cdot 22 - 15 \cdot 38) = 1.89$$

$$B = \frac{\Sigma w \, \Sigma wxy - \Sigma wx \, \Sigma wy}{\Delta} = \frac{1}{36}(9 \cdot 38 - 15 \cdot 22) = 0.33$$

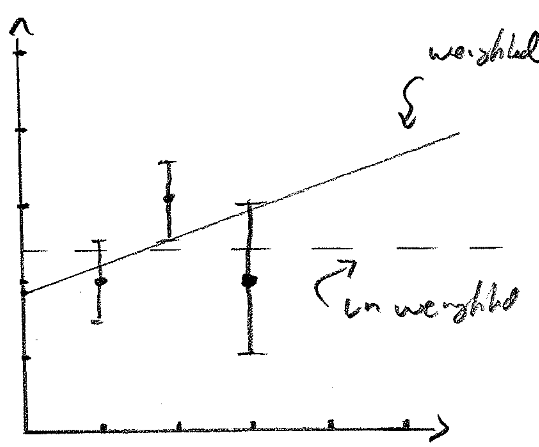
Unweighted

x	y	x^2	xy
1	2	1	2
2	3	4	6
3	2	9	6
6	7	14	14

$$\Delta = 3 \cdot 14 - 36 = 6$$

$$A = \frac{1}{6}(14 \cdot 7 - 6 \cdot 14) = 2.33$$

$$B = \frac{1}{6}(3 \cdot 14 - 6 \cdot 7) = 0$$



The 3rd point "pulls" the fit more in the unweighted case and flattens out the fit!