PHYS 399
HW #2

4.5/ We did this in class

a)
$$\sum (x_i - \bar{x})^2 = \sum x_i^2 - 2\bar{x}\sum x_i + \sum \bar{x}^2$$
$$= \sum x_i^2 - \frac{2}{N}\left[\sum x_i\right]^2 + N\bar{x}^2$$
$$= \sum x_i^2 - \frac{2}{N}\left[\sum x_i\right]^2 + N\frac{1}{N^2}\left(\sum x_i\right)^2$$
$$= \sum x_i^2 - \frac{1}{N}\left(\sum x_i\right)^2$$

b) $\bar{x} = 12$

$$\sum (x_i - \bar{x})^2 = 1^2 + 0^2 + 1^2 = 2$$
$$\sum x_i^2 - \frac{1}{N}\left(\sum x_i\right)^2 = 11^2 + 12^2 + 13^2 - \frac{1}{3}(36)^2$$
$$= 434 - 432 = 2$$

4.22/

a)

ℓ (m)	T (s)	g (m/s²)
0.573	1.521	9.78
0.611	1.567	9.82
0.732	1.718	9.79
0.837	1.835	9.81
0.950	1.952	9.84

b) $g = 4\pi^2 \ell / T^2$

$$\frac{\delta g}{g} = \sqrt{\left(\frac{\delta \ell}{\ell}\right)^2 + 4\left(\frac{\delta T}{T}\right)^2}$$
$$= \sqrt{(0.003)^2 + 4(0.002)^2}$$
$$= 0.005 \rightarrow 0.5\% \text{ relative}$$

c) Entering the values in my calculator gives

Mean: $9.810 \ m/s^2$

SD: $0.026 \ m/s^2 \Rightarrow \frac{.026}{9.81} = 0.27\%$

$\cong 0.3\%$

$\Rightarrow$ True variation in measured values is about (but a bit less) than the estimated error (0.5%).

d) $SDOM = \frac{0.026}{\sqrt{5}} = 0.012$

$$g = (9.810 \pm 0.012) \ m/s^2$$

Accepted $g = 9.796 \ m/s^2$

$\Delta = (1.4 \pm 1.2) \ cm/s^2 \Rightarrow 1.2\,\sigma$ discrepancy

Perfectly acceptable!

4.23)

$e = K\eta^{3/2}$ $\qquad \frac{\delta e}{e} = \frac{3}{2}\frac{\delta \eta}{\eta}$

$= \frac{3}{2}(0.4\%) = 0.6\%$

The systematic from η resulted in a 0.6% relative error on e, which likely dominates.

4.26

a)

V (V)	I (A)	R (Ω)
11.2	4.67	2.398
13.4	5.46	2.454
15.1	6.28	2.404
17.7	7.22	2.452

Here I have kept an extra digit in R.

I will also calculate δR directly from the observed variation in the derived result.

$$\bar{R} = 2.427 \qquad SD = 0.030 \qquad \text{(from Matlab) } (N-1)$$

$$\delta R_{ran} = \sigma_R / \sqrt{N} = \frac{0.030}{2} = 0.015$$

$$R = (2.427 \pm 0.015)\ \Omega$$

⇒ Needed extra digit after all!

b) Use 2% error on both V and I

$$R = V/I \qquad \frac{\delta R}{R} = \sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2}$$

$$\left(\frac{\delta R}{R}\right)_{syst} = \sqrt{2} \cdot 2\% = 2.8\%$$

$$\text{or} \quad \delta R_{syst} = 2.8\% \cdot 2.43\,\Omega = 0.068\ \Omega$$

$$R = [2.43 \pm 0.02\,(stat) \pm 0.07\,(syst)]\ \Omega$$

$$\delta R = \sqrt{(.015)^2 + (.068)^2} = 0.07\ \Omega \quad \underline{\text{total}}$$

5.6 ✓

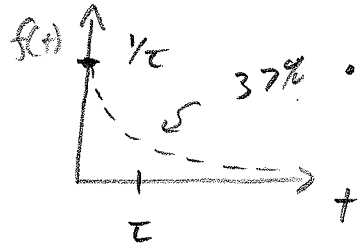
$$f(t) = \frac{1}{\tau} e^{-t/\tau}$$

$u = t/\tau$

$du = \frac{1}{\tau} dt$

First, check the normalization!

$$\int_0^\infty f(t)\, dt = \frac{\tau}{\tau} \int e^{-u}\, du = -e^{-u}\Big]_0^\infty = 1 \checkmark$$

a) 

37% of original

b) Ok, normalization given above (I jumped the gun)

c) $$\bar{t} = \int t\, f(t)\, dt = \frac{1}{\tau}\int_0^\infty t e^{-t/\tau} dt = \frac{1}{\tau}\tau^2 e^{-t/\tau}(-t/\tau - 1)\Big]_0^\infty$$

$$= [0 - \tau e^{-0} \cdot (-1)] = \tau$$

τ is the mean time $\bar{t}$

5.12 ✓

$$g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Assume $\mu = 0$ and drop the normalization

$$g(0) = e^{-0} = 1$$

$g(x) = 1/2$ ← half max

$$e^{-x^2/2\sigma^2} = \frac{1}{2}$$

$$-\frac{x^2}{2\sigma^2} = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$x^2 = 2\sigma^2 \ln 2$$

$$x = \sigma\sqrt{2\ln 2}$$ ← Full width is twice this

5.21

$\bar{X} = 5' \, 5\frac{1}{2}'' = 65.5$ in

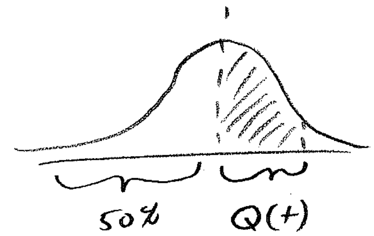
$\sigma_x = 2.5$ in

a) what is prob. of $x \geq 5'10'' = 70$ in ?

Note, this is single sided

$$\frac{70 - 65.5}{2.5} = 1.8$$

Easiest to use Table B



$$\text{Prob}(< 1.8\sigma) = \tfrac{1}{2} + Q(1.8)$$

$$= 0.5 + 0.464 = 96.4\%$$

So $P(> 1.8\sigma) = 1 - 96.4\% = 3.6\%$

2000 people $\rightarrow$ 72 greater than 70 inches

b) To get 144 (double) people, need

$$p = 7.2\%$$

$$1 - \underbrace{(0.5 + Q(t))}_{P(< t\sigma)} = 7.2\% = P(> t\sigma)$$

$Q(t) = 0.428$ Again table B

$$t = 1.46$$

$\Delta X = 1.46 \cdot 2.5$ in $= 3.65$ in.

Height = 69 inches (nearest half)

5.28

a) Want $\frac{\delta\sigma}{\sigma} = 0.3$

$$\frac{\delta\sigma}{\sigma} = \frac{1}{\sqrt{2(N-1)}} \qquad (5.46)$$

$$2(N-1) = \frac{1}{f^2} \qquad f = \frac{\delta\sigma}{\sigma}$$

$$N = \frac{1}{2f^2} + 1$$

a) $f = 0.3 \rightarrow N = \frac{1}{2(.3)^2} + 1 = 6.5 \sim 7$

b) $f = 0.1 \rightarrow N = \frac{1}{.02} + 1 = 51 \sim 50$

c) $f = 0.03 \rightarrow N = \frac{1}{2(.03)^2} + 1 = 556$

5.36

$X_A = 13 \pm 1$

$X_B = 15 \pm 1$

a) $\Delta = |X_A - X_B| = 2.0 \pm 1.4$ ← keep extra digit!

b) $\frac{2.0}{1.4} = 1.4\,\sigma$ discrepancy

From table: $1 - P(\pm 1.4\sigma) = (1 - .8385)$

$= 16\,\%$

Consistent at the 5% level.

Note here there is no reason to assume the discrepancy sign is relevant, so a double-sided probability is entirely appropriate.