Physics 417
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1. For a spin 1/2 particle, the operator that measures the component of spin along an axis $\vec{n}$ is (in a matrix notation) $\vec{S}\cdot\vec{n}$ where

$$2\,\vec{S}\cdot\vec{n} = \begin{pmatrix} n_z & n_x - i n_y \\ n_x + i n_y & -n_z \end{pmatrix} \quad . \tag{1}$$

Here $\vec{n}^2 = 1$. For $\vec{n} = \hat{z}$, the two eigenvectors of this operator are

$$u(+,\hat{z}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad u(-,\hat{z}) = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad . \tag{2}$$

Find the eigenvectors for a general $\vec{n}$. Your solution should be normalized to $\langle +,\vec{n}|+,\vec{n}\rangle = \langle -,\vec{n}|-,\vec{n}\rangle = 1$. A convenient choice of phases is to make the upper component of $u(+,\hat{z})$ real and positive and to make the lower component of $u(-,\hat{z})$ real and negative. Also, as a check, verify that $\langle +,\vec{n}|-,\vec{n}\rangle = 0$.

2. Consider the two particle state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+,\hat{z};-,\hat{z}\rangle - |-,\hat{z};+,\hat{z}\rangle\right) \quad . \tag{3}$$

Suppose that an observer Alice measures the component of spin along the $\vec{n}$ axis for the first particle and that another obeserver Bob measures the component of spin along the $\vec{n}$ axis for the second particle. The two possible results for Alice are $s_A = +1$ and $s_A = -1$. Similarly the two possible results for Bob are $s_B = +1$ and $s_B = -1$. The probability for Alice to get result s_A and for Bob to get results s_B is

$$\left|\langle s_A,\vec{n}; s_B,\vec{n}|\psi\rangle\right|^2 \quad . \tag{4}$$

Use your eigenvectors to compute this probability for $s_A = +1$, $s_B = +1$ and for $s_A = +1$, $s_B = -1$.

3. Griffiths problem 21.1.