

Physics 417
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Problems for 10 April, 2006

1. Suppose that you do a calculation in “natural units,” $c = \hbar = \epsilon_0 = 1$ and the answer for the time it takes for something to happen is $t = 2.0 \text{ m}$. What is t in seconds?
2. Suppose that you do a calculation using $c = \hbar = \epsilon_0 = 1$ and the answer for the energy of some state is $E = 2.0 \text{ m}^{-1}$. What is E in Joules? What is E in electron volts?
3. *Griffiths*, problem 4.17.
4. Here is another variation on the hydrogen atom: positronium. That’s an atom made of an electron e^- and an an antielectron e^+ . The e^+ has exactly the same mass as the e^- but the opposite charge. Thus the e^- and e^+ can make a bound state because of their electrostatic attraction. What is the binding energy $|E_1|$ of positronium? Please give an answer in terms of α and m_e and a numerical answer in eV. To do this, you need to solve

$$\left\{ -\frac{1}{2m_e} [\nabla_1^2 + \nabla_2^2] - \frac{e^2}{4\pi |\vec{x}_1 - \vec{x}_2|} \right\} \psi(\vec{x}_1, \vec{x}_2) = E\psi(\vec{x}_1, \vec{x}_2) . \quad (1)$$

You will want to change variables from $\{\vec{x}_1, \vec{x}_2\}$ to

$$\vec{r} = \vec{x}_1 - \vec{x}_2 \quad (2)$$

and

$$\vec{R} = \frac{1}{2}[\vec{x}_1 + \vec{x}_2] . \quad (3)$$

Note that R is the position of the center of mass of the positronium atom. Assume that

$$\frac{\partial}{\partial R_j} \psi(\vec{r}, \vec{R}) = 0 . \quad (4)$$

That corresponds to the atom having no net momentum. This should give you a standard hydrogen atom problem, for which you know the energy.

5. There is a quantum process in which the e^- and e^+ can annihilate to make two photons. Assume that the positronium was at rest before the annihilation. Then the two photons go off in opposite directions with the same energy. The entire energy $2m_e - |E_1|$ of the system goes into the photons. (Here I write m_e for the rest energy of an electron. With the c put back in, the rest energy is $m_e c^2$.) What is the wavelength of each photon? Does $|E_1|$ matter much here?