**Midterm exam**
PHYS 631, Quantum Mechanics
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D. E. Soper

There are five problems. Please answer them on separate sheets of paper provided. Please label each sheet with your name and the problem number. I am looking for not only an answer in the form of a number or a formula, but also a clear and concise explanation of your reasoning.

None of the problems requires a lot of computation, so if you are finding that your method of attack is requiring a lot of calculations, it might be best to go on to another problem and come back later to the one that was causing difficulties. Note that I set $\hbar = 1$.

1) Consider a quantum mechanical system based on three orthonormal basis vectors, $|1\rangle$, $|2\rangle$, $|3\rangle$. Suppose that the system is in the state $|\psi\rangle = \big(|1\rangle + |2\rangle\big)/\sqrt{2}$. Suppose further that there is a measurement that we can perform that will determine if the system is in the state $|\phi\rangle = \big(|2\rangle + |3\rangle\big)/\sqrt{2}$. What is the probability that the system prepared in state $|\psi\rangle$ will be found to be in state $|\phi\rangle$ if we make the measurement?

2) Consider a quantum mechanical system based on three orthonormal basis vectors, $|1\rangle$, $|2\rangle$, $|3\rangle$. Are the states

$$
\begin{aligned}
|A\rangle &= \frac{1}{\sqrt{3}}\Big(|1\rangle + |2\rangle + |3\rangle\Big) \\
|B\rangle &= \frac{1}{\sqrt{5}}\Big(|2\rangle + 2|3\rangle\Big) \\
|C\rangle &= \frac{1}{\sqrt{5}}\Big(2|1\rangle + |2\rangle\Big)
\end{aligned}
\tag{1}
$$

linearly independent? Why or why not?

3) Consider a quantum mechanical system based on three orthonormal basis vectors, $|1\rangle$, $|2\rangle$, $|3\rangle$. Find the eigenstates and eigenvalues of the hamiltonian

$$
H = \omega + \delta\Big(|2\rangle\langle 3| + |3\rangle\langle 2|\Big) \quad . \tag{2}
$$

(Here the $\omega$ multiplies the unit operator.)

4) Consider a particle that can move in one spatial dimension, with coordinate $x$. Consider the state $|\psi\rangle$ with spatial wave function

$$\langle x|\psi\rangle = \left[\frac{2a^3}{\pi}\right]^{1/2} \frac{1}{x^2 + a^2} \quad . \tag{3}$$

Let

$$|\phi\rangle = e^{-i\hat{p}b}|\psi\rangle \quad , \tag{4}$$

where $\hat{p}$ is the momentum operator and $b$ is a number. What is $\langle x|\phi\rangle$?

5) Consider a particle that can move in one spatial dimension, with coordinate $x$. The hamiltonian for the particle is

$$H = \frac{1}{2m}\,\hat{p}^2 + \frac{m\omega^2}{2}\,\hat{x}^2 \tag{5}$$

where $\hat{p}$ is the momentum operator and $\hat{x}$ is the position operator while $m$ is the mass of the particle and $\omega$ is a number that gives the strength of the potential. Using the Heisenberg picture to describe the dynamics of this system, what are the equations of motion for the momentum and position operators $\hat{p}(t)$ and $\hat{x}(t)$? You do not need to solve these equations to find $\hat{p}(t)$ and $\hat{x}(t)$ in terms of $\hat{p}(0)$ and $\hat{x}(0)$. I am merely looking for the differential equations that govern the time evolution of the operators.