

Hints for problems for 14 October¹

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1 “Vectors” notes exercise 1.5

This amounts to solving the matrix eigenvalue problem

$$\begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = 2\lambda \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} . \quad (1)$$

By solving the equation $\det(A - 2\lambda) = 0$, where A is the matrix in Eq. (1), we find that the eigenvalues are $\lambda = \pm 1/2$. Then the eigenvectors are

$$\begin{aligned} |\vec{S} \cdot \vec{n}, +\rangle &= \cos(\theta/2) |S_z, +\rangle + \sin(\theta/2) |S_z, -\rangle , \\ |\vec{S} \cdot \vec{n}, -\rangle &= -\sin(\theta/2) |S_z, +\rangle + \cos(\theta/2) |S_z, -\rangle . \end{aligned} \quad (2)$$

Notice how simple the result is when expressed in terms of $\sin(\theta/2)$ and $\cos(\theta/2)$. It is evident that these vectors are normalized and orthogonal to each other.

2 Sakurai problem 1.13

After the first S.G. apparatus, the system is in the state $|S_z, +\rangle$. After the second S.G. apparatus, the system is in the state $|\vec{S} \cdot \vec{n}, +\rangle$. The probability that an atom makes it through the second S.G. apparatus is

$$P_2 = |\langle \vec{S} \cdot \vec{n}, + | S_z, + \rangle|^2 = \cos^2(\beta/2) . \quad (3)$$

After the third S.G. apparatus, the system is in the state $|S_z, -\rangle$. The probability that an atom arriving at the third S.G. apparatus makes it through is

$$P_3 = |\langle S_z, - | \vec{S} \cdot \vec{n}, + \rangle|^2 = \sin^2(\beta/2) . \quad (4)$$

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The probability that an atom arriving at the second S.G. apparatus makes it through to the end is

$$P_2 P_3 = \cos^2(\beta/2) \sin^2(\beta/2) = \frac{1}{4} \sin^2(\beta) . \quad (5)$$

This tends to zero when $\beta \rightarrow 0$ because then the atom almost never makes it through the third S.G. apparatus. It tends to zero when $\beta \rightarrow \pi$ because then the atom almost never makes it through the second S.G. apparatus. It is biggest when $\beta = \pi/2$. For that choice, $P_2 = 1/2$ and $P_3 = 1/2$, so $P_2 P_3 = 1/4$.