Physics 661

Particle Physics Phenomenology

October 25, 2001

Invariance Principles and Conservation Laws

- Outline
    - Translation and rotation
    - Parity
    - Charge Conjugation
    - Baryon and lepton conservation
    - CPT Theorem
    - CP violation and T violation
    - Isospin symmetry

Conservation Rules

		Interaction	
Conserved quantity	strong	EM	weak
energy-momentum	yes	yes	yes
charge			
baryon number			
lepton number			
CPT	yes	yes	yes
P (parity)	yes	yes	no
C (charge conjugation parity)	yes	yes	no
CP (or T)	yes	yes	10^{-3} violation
I (isospin)	yes	no	no

Discrete and Continuous Symmtries

- Space-time symmetries
- Lorentz transformations
- Poincare transformations
  - combined space-time translation and Lorentz T.
- Discrete symmetries
  - cannot be built up from succession of infinitesimally small transformations
- Dynamical symmetries
- Internal symmetries

Translation and Rotation

- Invariance of the energy of an isolated physical system under space tranlations leads to conservation of linear momentum

- Invariance of the energy of an isolated physical system under spatial rotations leads to conservation of angular momentum

Parity

- Spatial inversion
  - $(x,y,z) \rightarrow (-x,-y,-z)$
  - discrete symmetry
- $P\,\psi(r) = \psi(-r)$
  - P is the parity operator
- $P^2\,\psi(r) = P\,\psi(-r) = \psi(r)$,
  - therefore $P^2 = 1$ and the parity of an eigen-system is 1 or -1

Parity

- Example, the spherical harmonics:

$$\boxed{Y_L^M}$$

- The spherical harmonics describe a state in a spherically symmetric potential with definite ang. momentum

$$Y_1^0 = \sqrt{\tfrac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\tfrac{3}{8\pi}}\sin\theta e^{i\phi}$$

$$Y_2^0 = \sqrt{\tfrac{5}{4\pi}}(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2})$$

$$Y_2^1 = -\sqrt{\tfrac{15}{8\pi}}\sin\theta\cos\theta e^{i\phi}$$

$$Y_2^2 = \tfrac{1}{4}\sqrt{\tfrac{15}{2\pi}}\sin^2\theta e^{2i\phi}$$

- $p = (-1)^L$

Parity

- Parity is a <u>multiplicative</u> quantum number

- Composite system:
  - parity is equal to product of the parts
  - eg. two pions in an angular momentum L state

$$P = P(\pi_1) \bullet P(\pi_2) \bullet (-1)^L$$

- Intrinsic parity of particles:
  - proton and neutron (+1 by definition, could be -1)
  - pion (-1, measured)

Pion spin and parity

- 1. Spin of the charged pion
  - $p + p \leftrightarrow \pi^+ + d$ $\quad (|M_{if}|^2 = |M_{fi}|^2)$
  - then, "detailed balance" gives:

    $\sigma(pp \rightarrow \pi^+ d) \sim (2s_\pi + 1)(2s_d + 1)\, p_\pi^2$

    $\sigma(\pi^+ d \rightarrow pp) \sim 1/2\, (2s_p + 1)^2\, p_p^2$

    (1/2 because protons are identical)
  - Measurements show $s_\pi = 0$

- 2. Spin of the neutral pion
  - $\pi^0 \rightarrow \gamma\gamma$ shows it must be 0

Pion spin and parity

- 2. Spin of the neutral pion
  - $\pi^0 \rightarrow \gamma\gamma$ shows it must be 0
  - along the flight path of the γs in the π^0 rest frame, the total photon spin (S_z) must be 0 or 2
  - If $S_\pi = 1$, then S_z must be 0
  - If $S_z = 0$, the two-photon amplitude must behave as $P_l^{m=0}(\cos\ \theta)$, which is antisymmetric under interchange (=180^0 rotation)
  - This corresponds to two right or left circularly polarized photons travelling in opposite directions, which violates Bose symmetry
  - Therefore, $S_\pi = 0$ or $S_\pi \geq 2$ (which is ruled out by production statistics)

Parity

- Parity of the charged pion
  - the observation of the reaction

    $\pi^- + d \rightarrow n + n$
  - is evidence that the charged pion has p = -1
- Capture takes place from an s-state (S_d=1)
  - (X-ray emissions following capture)
- In the two neutron system, L+S must be even by the antisymmetric requirement on identical fermions
  - thus, $^3P_1$ state with $p = (-1)^L = -1$
  - since initial state is $p_d p_\pi = (+1)\ p_\pi$, $p_\pi = -1$

Parity

- Parity of the neutral pion

  $\pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-)$ (rare decay of the pion)

  - the planes of the pairs follow the E vectors of the internally converted photons ($\pi^0 \rightarrow \gamma\gamma$)
  - even system of two photons bosons

    $A \sim (\mathbf{E_1} \bullet \mathbf{E_2})$ (P=+1)

    or $A \sim (\mathbf{E_1} \times \mathbf{E_2}) \bullet \mathbf{k}$ (P=-1)

  - Intensities or rates $\sim |A|^2$

    $|A|^2 \sim \cos^2 \phi$

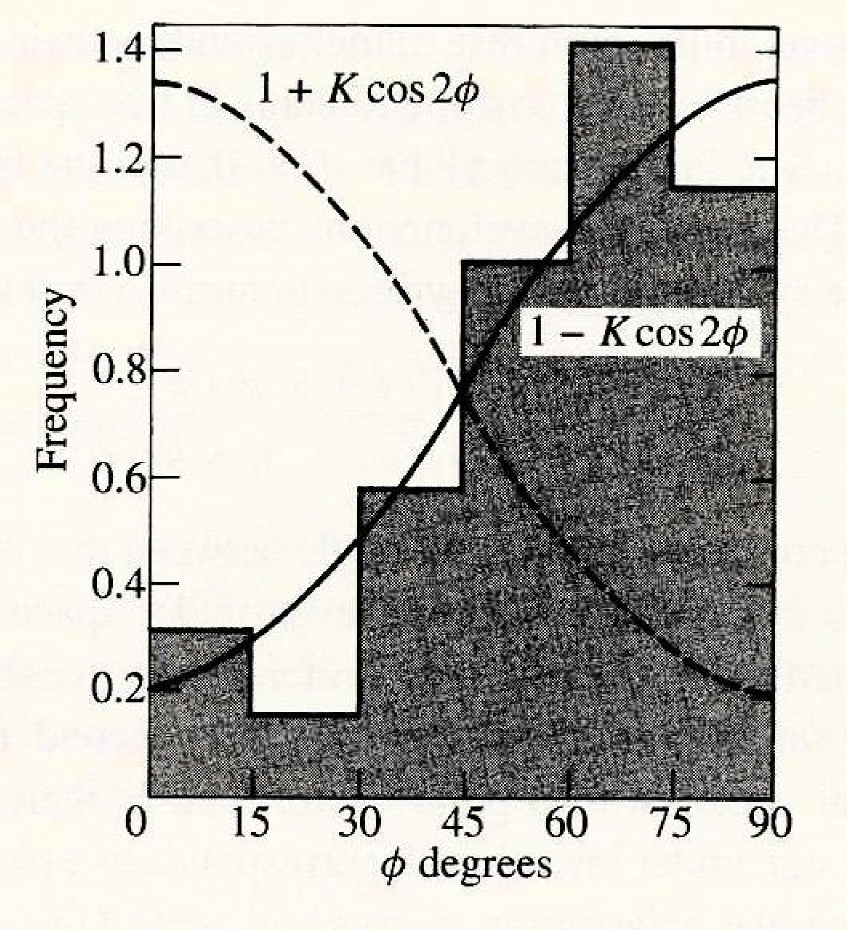
    or $|A|^2 \sim \sin^2 \phi$

Parity

- Parity of the neutral pion

  $\pi^0 \rightarrow (e^+ + e^-) + (e^+ + e^-)$

  scalar(0^+) would follow dashed line, while pseudoscalar (0^-) would follow solid line



Parity of particles and antiparticles

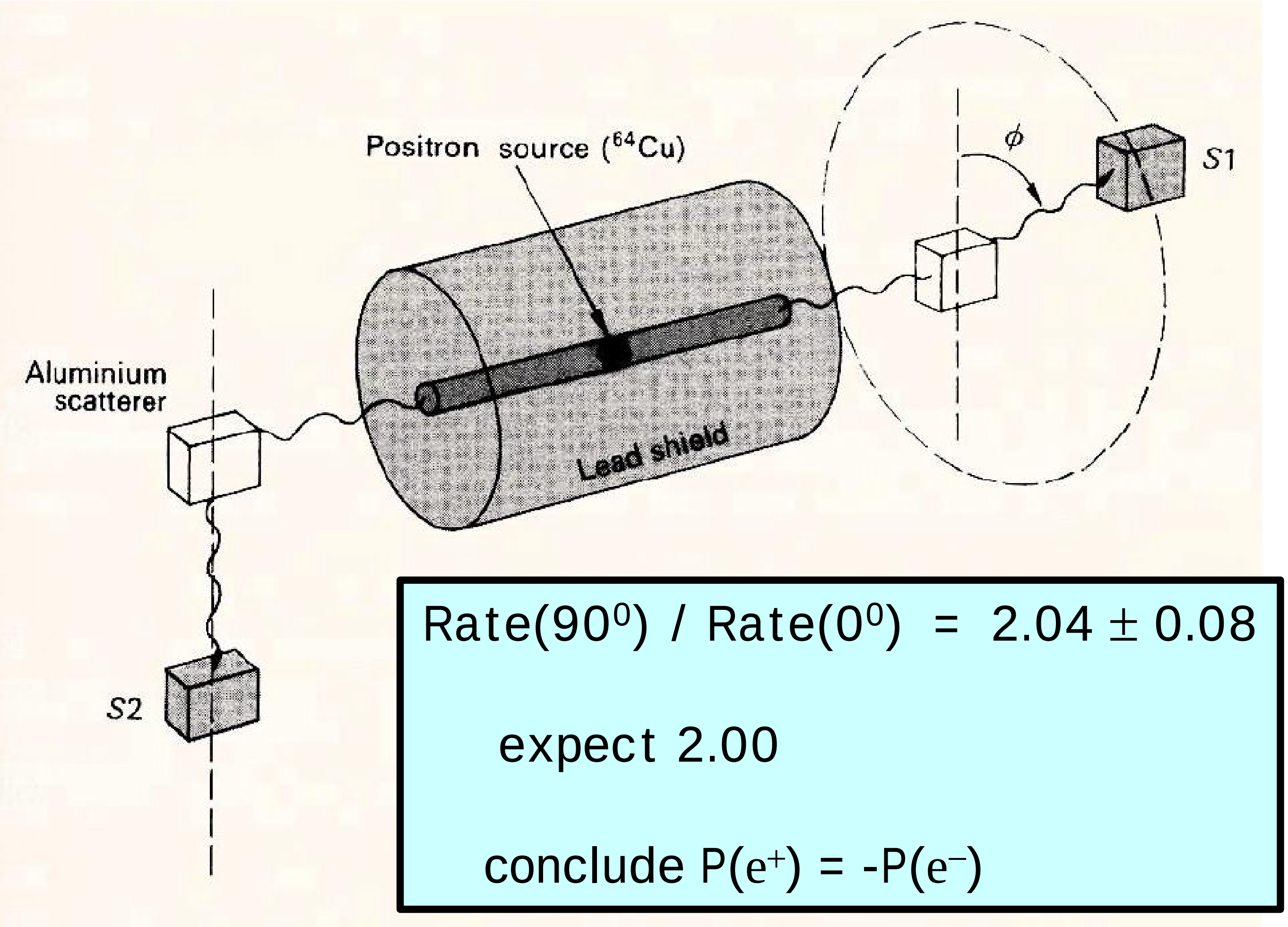
- Dirac theory required fermions and antifermions to have opposite intrinsic parity
- This was checked in decays of the spin singlet ground state of positronium

$$e^+e^- \, (^1S_0) \rightarrow 2\,\gamma$$

$$P = (+1)\,(-1)\,(-1)^0 = -1$$

- This is exactly the case of the π^0 decay
  - the photon polarizations must have a $\sin^2\phi$ form

Parity of particles and antiparticles



Rate(90^0) / Rate(0^0) = 2.04 ± 0.08

expect 2.00

conclude $P(e^+) = -P(e^-)$

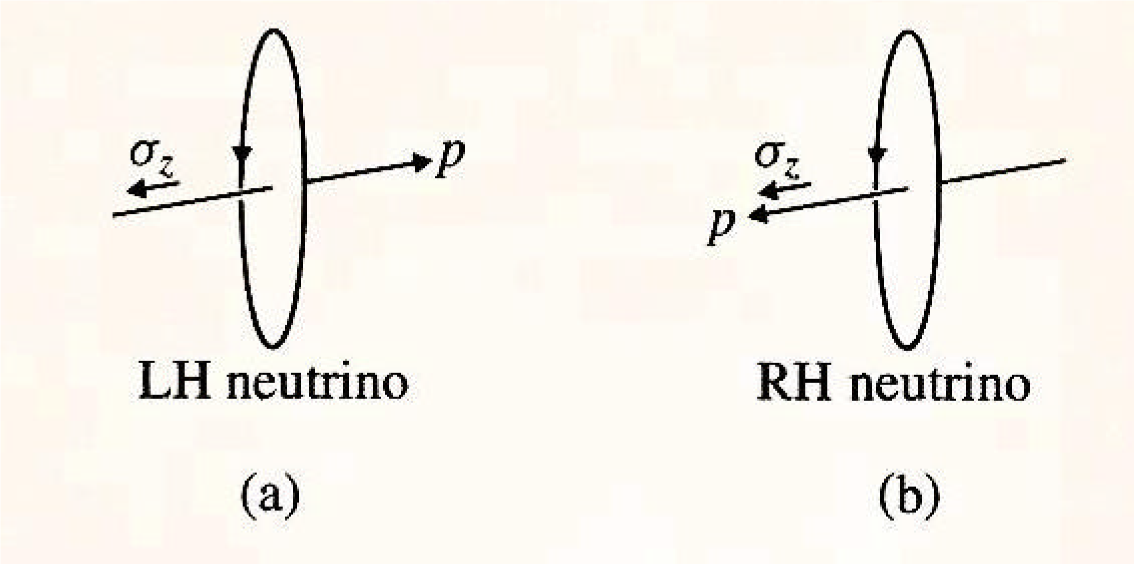
Parity of particles and antiparticles

- Two-particle systems
  - Fermion-antifermion
    - $p = (-1)^{L+1}$
    - eg, pion which is q-antiq with L=0 has p = -1
  - Boson-antiboson
    - same parity
    - $p = (-)^{L}$
    - eg. $\rho \to \pi^{+}\ \pi^{-}$

      $1^{-} \to 0^{-}\ 0^{-}$, L=1

Tests of parity conservation

- Strong and EM interactions conserve parity, but weak interactions do not



- LH neutrino observed, but RH neutrino is not
  - this is a maximally violated symmetry

Tests of parity conservation

- In interactions dominated by the strong or the EM interaction, some parity violation may be observed due to the small contribution of the weak int. to the process

$$H = H_{strong} + H_{EM} + H_{weak}$$

- In nuclear transitions, for example, the degree of parity violation will be of the order of the ratio of the weak to the strong couplings ($\sim 10^{-7}$)

Tests of parity conservation

- Examples of nuclear transitions
  - fore-aft asymmetry in gamma emission

$$^{19}\mathrm{F}^{*} \rightarrow {}^{19}\mathrm{F} + \gamma \text{ (110 keV)}$$

$$J^{P}\text{:} \quad 1/2^{-} \rightarrow 1/2^{+}$$

$$\Delta \sim 10^{-4}$$

  - very narrow decay

$$^{16}\mathrm{O}^{*} \rightarrow {}^{12}\mathrm{C} + \alpha$$

$$J^{P}\text{:} \quad 2^{-} \rightarrow 2^{+}$$

$$\Gamma = 10^{-10} \text{ eV} \quad (\text{note } {}^{16}\mathrm{O}^{*} \rightarrow {}^{16}\mathrm{O} + \gamma, \Gamma = 3 \text{ x } 10^{-3} \text{ eV})$$